# N and Δ exchange amplitudes in πN backward scattering



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# *N* and Δ exchange amplitudes in πN backward scattering

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**Abstract.** The dual absorptive model is applied to πN backward scattering in detail and through the fits to the available data, the $N(I_u = \frac{1}{2})$ and $\Delta(I_u = \frac{3}{2})$ exchange amplitudes are studied. The interaction radius $R$ and the slope parameter $B$ for the $N$ amplitudes are found to be consistent with those of the forward scattering, while the slope parameter $B$ for the Δ amplitude is found to be rather large. This inconsistency is attributed to the incorrect form of the Re Δ as given by the model. Possible mechanisms to produce the correct Re Δ are given.

## 1. Introduction

In recent years, a number of phenomenological models (Berger and Fox 1970) have been proposed to account for the observed features of high energy πN backward differential cross sections and polarizations (Boright *et al* 1970, Orear *et al* 1968, Owen *et al* 1969, Aoi *et al* 1971, 1972, Bradamante *et al* 1973) but none of the Regge pole and Regge pole plus various cut models were able to explain the new data on backward $\pi^-$p polarization at 6 GeV/$c$ (Aoi *et al* 1973). In view of this puzzling situation, Berger and Olsson (1972) had determined, in a model independent way, the $I_u = \frac{1}{2}$ and $I_u = \frac{3}{2}$ exchange cross sections and interference terms from $\pi^{\pm}$p and CEX backward differential cross sections. Since then, a number of amplitude analyses based on the dual absorptive model (DAM) (Davier and Harari 1971, Harari 1971a,b) have been made (Tin Maung Aye 1972, Ferro Fontan 1972, Takahashi and Kohsaka 1973†). We have shown in our previous paper (Tin Maung Aye 1972) that the DAM, with a few plausible assumptions, could explain all the qualitative features of the backward cross sections and polarizations at 6 GeV/$c$‡. Similar analysis had been made by Ferro Fontan (1972), but with a different phase convention for the flip amplitudes. No specific parameters had been quoted and his solution for the imaginary part of the $I_u = \frac{3}{2}$ exchange amplitude has undesirable double zeros around $u = -0{\cdot}4$. In a recent amplitude analysis of Takahashi and Kohsaka (1973), the solutions for $I_u = \frac{1}{2}$ exchange amplitudes are similar to, but for $I_u = \frac{3}{2}$ exchange amplitude is different from, that of Ferro Fontan. In particular, they pointed out that the real part of the $I_u = \frac{3}{2}$ flip amplitude must have a single zero near $u = -0{\cdot}6$.

In this paper, we examine our previous model in some detail paying particular attention to the phases, the radius and the slope parameters of the two amplitudes.

† We received this preprint while our work was being done.
‡ Tin Maung Aye (1972) used an opposite relative sign between flip and non-flip amplitudes.

## 2. The model

As given in some detail in our previous paper (Tin Maung Aye 1972), the model requires the imaginary parts to be dominated by the most peripheral partial waves within the interaction radius $R \simeq 1$ fm, and we have for the real and the imaginary parts of the $I_u = \frac{1}{2}$ (denoted by $N$) and $I_u = \frac{3}{2}$ (denoted by $\Delta$) exchange amplitudes, the following parametrization (Davier and Harari 1971, Harari 1971a,b):

$$\begin{aligned} \operatorname{Im} N_{\Delta\lambda} &= A^N_{\Delta\lambda} \exp(B^N_{\Delta\lambda} u') J_{\Delta\lambda}(R^N \sqrt{-u'}), \\ \operatorname{Re} N_0 &= -A^N_0 \exp(B^N_0 u') J_0(R^N \sqrt{-u'}) \cot \tfrac{1}{2}(\pi \bar{\alpha}_N), \\ \operatorname{Re} N_1 &= A^N_1 \exp(B^N_1 u') C_N \text{ (unknown)} \end{aligned} \tag{1}$$

and

$$\begin{aligned} \operatorname{Im} \Delta_{\Delta\lambda} &= A^\Delta_{\Delta\lambda} \exp(B^\Delta_{\Delta\lambda} u') J_{\Delta\lambda}(R^\Delta \sqrt{-u'}), \\ \operatorname{Re} \Delta_0 &= -A^\Delta_0 \exp(B^\Delta_0 u') C_\Delta \text{ (unknown).} \\ \operatorname{Re} \Delta_1 &= A^\Delta_1 \exp(B^\Delta_1 u') J_1(R^\Delta \sqrt{-u'}) \tan \tfrac{1}{2}(\pi \bar{\alpha}_\Delta) \end{aligned} \tag{2}$$

where $\Delta\lambda = 0$ and 1 denotes the $s$ channel helicity non-flip and flip, $A$ and $B$ are the magnitude and the slope parameters respectively. $C_N$, $C_\Delta$ are the two unknown functions which serve as inputs to our analysis. $u'$ is defined as $u' \equiv u - u_0$ with $u_0 = (m^2 - \mu^2)^2/s$ and $\bar{\alpha} \equiv \alpha - \frac{1}{2}$. The normalizations and the phases of the amplitudes at $u' = 0$ are given by the Regge model.

The differential cross sections ($\sigma$), polarization ($P$) and the interference terms ($D$) are given by (Barger and Olsson 1972, Barger *et al* 1972)

$$\sigma = \sum_{\Delta\lambda} |G_{\Delta\lambda}|^2$$

$$\sigma p = 2 \operatorname{Im}(G_1 G_0^*) \qquad (i = +, -, 0)$$

where for

$$\begin{aligned} &\pi^+ p \to p\pi^+, && G^+ = \tfrac{1}{3}(2N + \Delta) \\ &\pi^- p \to p\pi^-, && G^- = \Delta \\ &\pi^- p \to n\pi^0, && G^0 = (\sqrt{2}/3)(N - \Delta). \end{aligned} \tag{3}$$

Alternatively, the set of observables $(\sigma_+, \sigma_-, \sigma_0)$ can be replaced by a new set

$$\begin{aligned} \sigma_N &= \sum_{\Delta\lambda} |N_{\Delta\lambda}|^2 \\ \sigma_\Delta &= \sum_{\Delta\lambda} |\Delta_{\Delta\lambda}|^2 \\ D &= \sum_{\Delta\lambda} \operatorname{Re}(N^*_{\Delta\lambda} \Delta_{\Delta\lambda}) \end{aligned} \tag{4}$$

and the relations between the two are

$$\begin{aligned} \sigma_\Delta &= \sigma_- \\ \sigma_N &= \tfrac{1}{2}[3(\sigma_+ + \sigma_0) - \sigma_-] \\ D &= \tfrac{1}{4}[3(\sigma_+ - 2\sigma_0) + \sigma_-]. \end{aligned} \tag{5}$$

We have used all possible data on $\sigma_{\pm,0}$ for $P_{\text{lab}} > 3$ GeV/$c$ (Boright *et al* 1970, Orear *et al* 1968, Owen *et al* 1969), and 6 GeV/$c$ data on polarization $P_{\pm}$ (Aoi *et al* 1971, 1972, 1973, Bradamante *et al* 1973).

In equation (1), not all the parameters are free and we fix some of them as follows.

We start by looking at the 6 GeV/$c$ data. The parameters $A_0^N$ and $A_0^\Delta$ are fixed by the backward peaks ($u = 0$) of $\sigma_N$ and $\sigma_\Delta$. We get

$$A_0^N \simeq 9{\cdot}2; \qquad A_0^\Delta \simeq 1{\cdot}7.$$

The radius $R$ is assumed to be independent of helicity change $\Delta\lambda$ and the $u$ channel isospin†. We take $R_{\Delta\lambda}^N \simeq R_{\Delta\lambda}^\Delta \simeq 1$ fm and exact values are obtained by requiring $J_0(R^N\sqrt{-u'})$ and $J_1(R^\Delta\sqrt{-u'})$ to vanish at $u$ values where poles are developed in $\cot\frac{1}{2}(\pi\bar{\alpha}_N)$ and $\tan\frac{1}{2}(\pi\bar{\alpha}_\Delta)$, where

$$\alpha_N = -0{\cdot}45+0{\cdot}97u,$$

$$\alpha_\Delta = 0+0{\cdot}95u.$$

Therefore Re $N_0$ and Re $\Delta_1$ are nonzero near $u \simeq -1{\cdot}0$ and $u \simeq -0{\cdot}5$, respectively. This is a very severe restriction on the DAM amplitudes and plays a decisive test of the model in spite of the presence of other unknown real parts.

We are therefore left with three free parameters (at every $s$) $A_1$, $B_0$ and $B_1$ for each $N$ and $\Delta$ amplitude. There are two possibilities of having the same or opposite relative signs between the flip and the non-flip amplitudes, corresponding to $A_1 > 0$ to $A_1 < 0$, respectively. In each case, the unknown functions $C_N$ and $C_\Delta$ can be determined as follows.

### 2.1. $C_N$

The isospin bounds on the polarization ($P_N$) due to $N$ exchange amplitudes (Barger and Olsson 1972, Barger *et al* 1972)

$$P_{N1} \leqslant P_N \leqslant P_{Nu}$$

where

$$P_{N1} = \frac{1}{4}\left[3\left(\frac{\sigma_+}{\sigma_N}(1+P_+)\right)^{1/2} - \left(\frac{\sigma_-}{\sigma_N}(1+P_-)\right)^{1/2}\right]^2 - 1$$

$$P_{Nu} = 1-\frac{1}{4}\left[3\left(\frac{\sigma_+}{\sigma_N}(1-P_+)\right)^{1/2} - \left(\frac{\sigma_-}{\sigma_N}(1-P_-)\right)^{1/2}\right]^2$$

can be converted by using equation (1) to the isospin bounds on $C_N$, namely

$$C_{N1} \leqslant C_N \leqslant C_{Nu}.$$

The bounds on $C_N$, of course, depend on the parameters $A^N$ and $B^N$. In both cases of $A_1^N \gtrless 0$, we normalize $C_N$ at $u = u_0$ according to the Regge pole phase and thus it is expected to be small near $u = u_{\text{d}}$ ($u_{\text{d}} = -0{\cdot}15$). As a start, we take $C_N = 0$ at $u \simeq u_{\text{d}}$ and estimate $A_1^N$ from the value $\sigma_N(u_{\text{d}})$ for $B_1^N \simeq 1$. With these values for $A$ and $B$, an estimate of the bounds on $C_N$ can be obtained. The inset of figure 3($b$) shows $C_N$ bounds for $A_1^N > 0$. We can then take an input $C_N$, as shown in the figure, for the detail fitting.

† This assumption is in the same spirit as in the forward case (Tin Maung Aye 1972).

### 2.2. $C_\Delta$

Since we have both the $\sigma_-$ and $P_-$ data at 6 GeV/$c$ for fitting, we parametrize $C_\Delta$ with two additional free parameters $a$ and $b$, being normalized approximately by the Regge pole model at $u = u_0$. We write

$$C_\Delta = 0{\cdot}9 + au' + bu'^2.$$

We then proceed to fit simultaneously the data for $\sigma_N$, $D$, $\sigma_-$ and $P_-$ all at 6 GeV/$c$, first by taking $A_1 < 0$, which corresponds to the phase convention used in our previous qualitative analysis (Tin Maung Aye 1972). The fits we obtained are rather bad, in particular for $D$. We also get unreasonable parameter values for $B$.

In the case of $A_1 > 0$, which is the phase convention used by Ferro Fontan (1972) and by Takahashi and Kohsaka (1973), we obtain quite good fits for all $u \gtrsim -0{\cdot}6$, which are shown in figures 1($b$), 2, 4($b$), 5($b$). We obtain an overall $\chi^2$/ND (ND is the number of degrees of freedom) of about 1·5. The corresponding real and imaginary parts of $N$ and $\Delta$ amplitudes are displayed in figures 3($a$–$d$), and the calculated cross sections and polarizations are shown in figures 4($a$, $c$) and figures 5($a$, $c$), respectively. We now examine our results for the $N$ and $\Delta$ amplitudes.

### 2.3. *The N amplitude*

From the fits we obtain the values for the parameters $A_1^N$, $B_0^N$ and $B_1^N$ as tabulated in table 1. We find that $B_0^N \simeq B_1^N$ and the magnitude is quite consistent with those obtained in the DAM analysis for the forward scattering (Matthews 1972). In figures 3($a$, $b$) the real and the imaginary parts of the $N_0$ and $N_1$ amplitudes are shown. These are qualitatively similar to, but much smoother than, the corresponding amplitudes obtained by Ferro Fontan (1972) and Takahashi and Kohsaka (1973). Im $N_0$ and Im $N_1$ are clearly

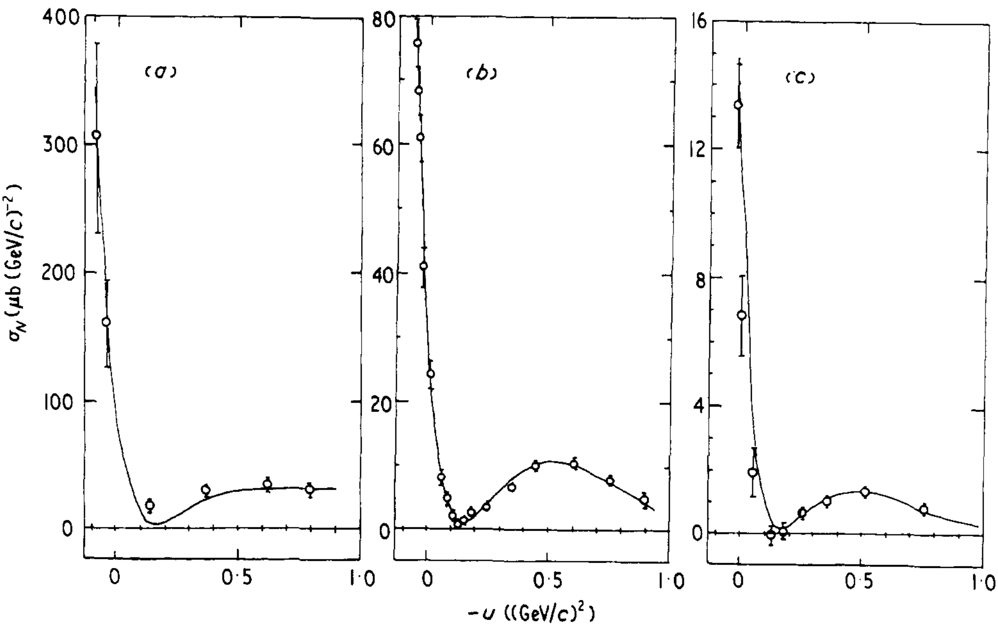


**Figure 1.** Fits to $\sigma_N$ at ($a$) 4 GeV/$c$, ($b$) 6 GeV/$c$ and ($c$) 10 GeV/$c$.

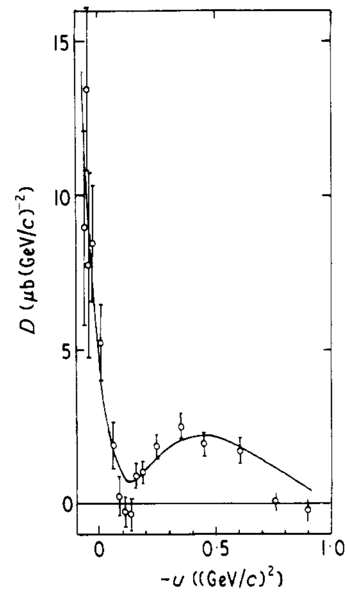


**Figure 2.** Fit to the interference term $D$ at 6 GeV/$c$.

peripheral, since the width of the corresponding impact parameter distribution ($\sim\sqrt{B}$) is much smaller than the interaction radius $R$. However, the real parts are obviously not peripheral. Further improvement to the fits by varying the form of $C_N$ is not necessary for reasons concerned with the Δ amplitudes discussed later.

In view of this encouraging support for the model, we examine the $s$ behaviour of the parameters $R_N$ and $B_N$ by using the available data. From the fits to the backward peaks of $\sigma_N$ at different $s$, we obtain†

$$A_0^N \simeq (283 \pm 25)s^{-1\cdot 4}$$

and we assume‡ exactly similar $s$ dependence for $A_1^N$, the normalization being given by the $A_1^N$ value at 6 GeV/$c$. Thus we obtain

$$A_1^N \simeq (77\cdot 4 \pm 8\cdot 0)s^{-1\cdot 4}.$$

We then proceed to fit the data on $\sigma_N$ having, at each energy, $B_0^N = B_1^N$ and $R^N$ as free parameters. The values obtained are shown in table 1. As in the forward scattering (Matthews 1972), $R^N$ is found to be nearly constant in $s$ and the slope parameter $B^N$ increases approximately like ln $s$.

### 2.4. *The Δ amplitude*

The results for Δ amplitudes are quite opposite to those for $N$ amplitudes. The parameter values obtained are given in table 1. Although we have $B_0^\Delta \simeq B_1^\Delta$, the values obtained are rather large and the result is a very poor peripheral nature of the impact parameter

† We have introduced an overall normalization error of about 10 %.

‡ Incomplete data and the smallness of $\sigma_N$ near $u = u_d$ force us to assume this form.

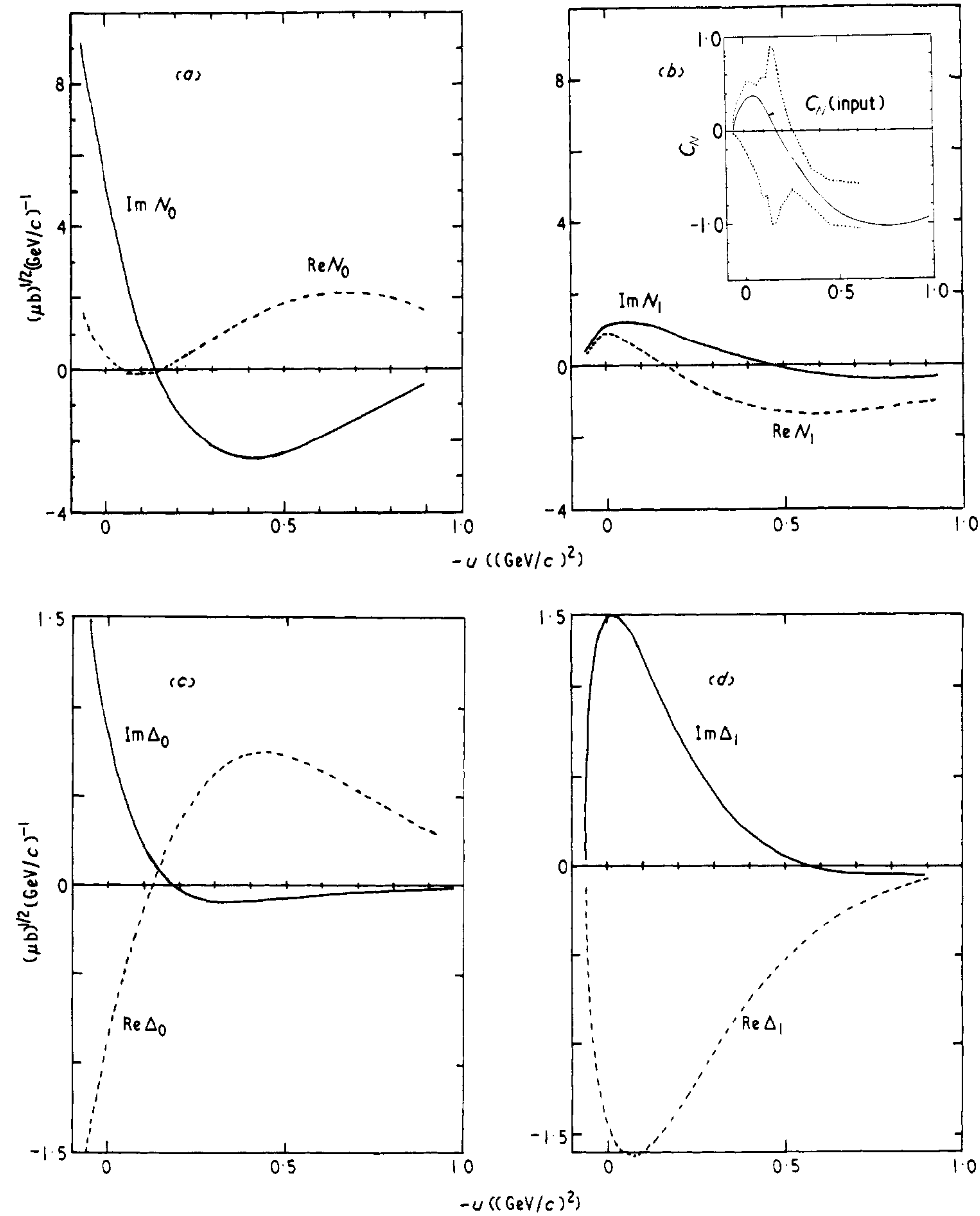


**Figure 3.** The real and the imaginary parts of $N$ and $\Delta$ amplitudes at 6 GeV/$c$. Inset of figure 3($b$) shows the isospin bounds on our input function $C_N$.

distributions of the Im $\Delta_0$ and Im $\Delta_1$. The fits to $D$, $\sigma_-$ and $P_-$ as shown in figures 2, 4($b$), 5($b$) are reasonably good down to $u \simeq -0{\cdot}6$, showing approximate correctness of the phases of the $\Delta$ amplitudes. The real and the imaginary parts of $\Delta_0$ and $\Delta_1$ are displayed in figures 3($c$, $d$). Our amplitudes are qualitatively similar to those of Ferro Fontan, but without the double zero in Im $\Delta_0$ near $u \simeq -0{\cdot}4$. However, our Im $\Delta_0$ is also very much damped after the first zero. For the region $u < -0{\cdot}6$, the forms of

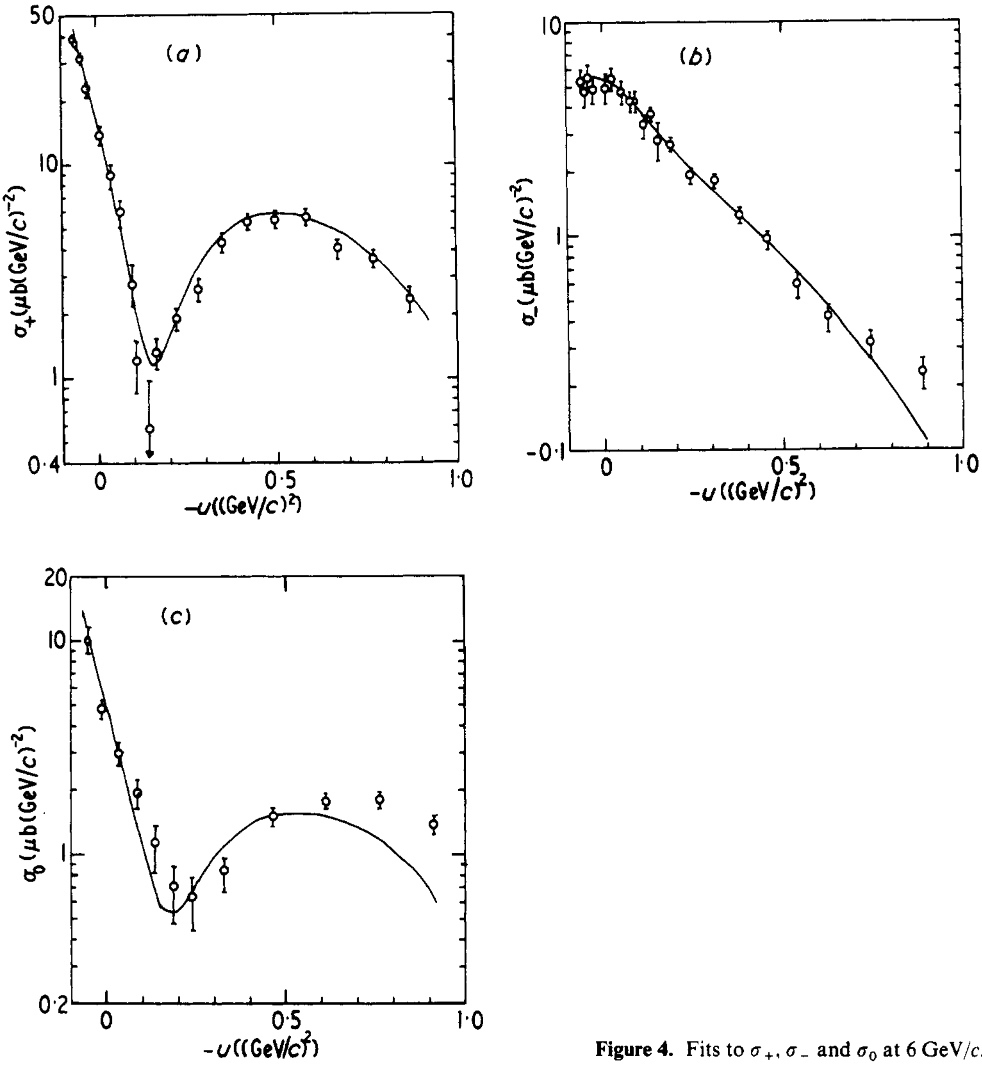


**Figure 4.** Fits to $\sigma_+$, $\sigma_-$ and $\sigma_0$ at 6 GeV/$c$.

either one or both of Re $\Delta_0$ and Re $\Delta_1$ are clearly wrong. The effects are qualitatively visible in the fits to $\sigma_-$, $D$ and $\sigma_0$. Also we cannot reproduce the sharp peak in $P_-$ before the zero at $u \simeq -0{\cdot}6$. This sharpness of the peak before the zero is rather suggestive of the fact that both Im $\Delta_1$ and Re $\Delta_1$ are zero at $u \simeq -0{\cdot}5$, whereas in our model the zero is produced by the 180° phase difference between $\Delta_0$ and $\Delta_1$.

A more detailed look at the individual amplitude revealed that Re $\Delta_1 \sim J_1(R\sqrt{-u'})$ $\cot \frac{1}{2}(\pi\bar{\alpha}_\Delta)$ must be very small or zero near $u \simeq -0{\cdot}5$, the solution which Takahashi and Kohsaka obtained in their amplitude analysis. This form of Re $\Delta_1$ is rather hard to realize in a simple DAM. However, as pointed out by Harari and Schwimmer (1972), particular asymptotic energy behaviour of the parameters $A$, $B$ and $R$ could lead to non-Regge-pole-type complex $J$-plane structures (eg, fixed poles, a complex conjugate pair of moving branch points, etc) and the resulting real parts do not have poles due to the factor like $\cot \frac{1}{2}(\pi\bar{\alpha}_\Delta)$ (at $\bar{\alpha}_\Delta = -1$). In such a case, the real parts also have zero structures around $u$ (or $t$) values where the corresponding imaginary parts vanish. This kind of

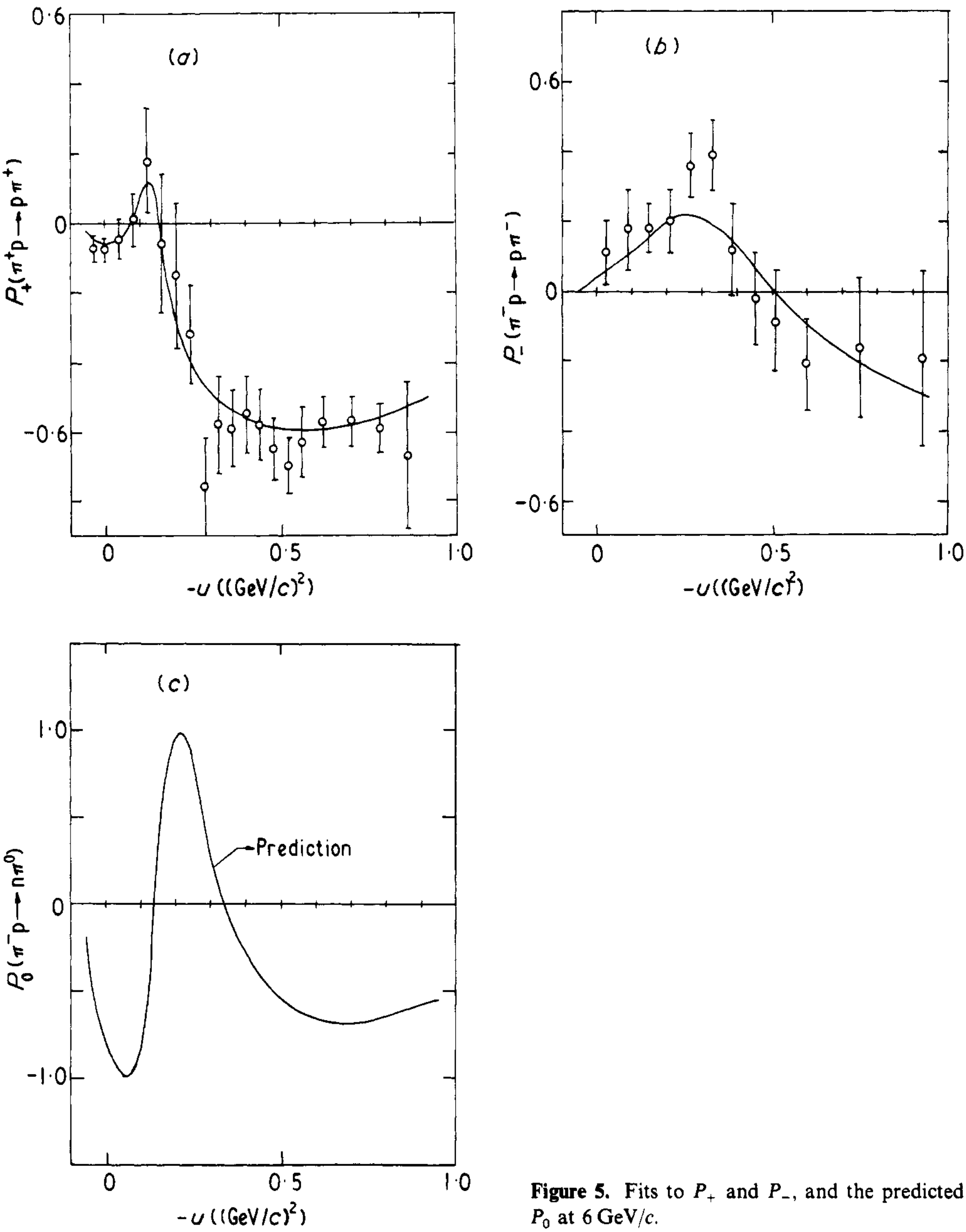


**Figure 5.** Fits to $P_+$ and $P_-$, and the predicted $P_0$ at 6 GeV/$c$.

mechanism is exactly what is needed for both $\mathrm{Re}\,\Delta_0$ and $\mathrm{Re}\,\Delta_1$. The corresponding consequences on the peripherality, the dip positions (the cross-over position) and the shrinkage of the differential cross section will be rather different from that of the DAM. These points will be considered elsewhere. Strong absorption models are also consistent with our conclusion about $\mathrm{Re}\,\Delta_0$ and $\mathrm{Re}\,\Delta_1$, since these models require both the imaginary and the real parts to be peripheral†.

† To be consistent with correct pole extrapolation, we need to multiply all our amplitudes by $-1$.

**Table 1.**

| | $N$ | | | $\Delta$ | Units |
|---|---|---|---|---|---|
| $P_{\text{lab}}$ | 4 | 6 | 10 | 6 | (Gev/$c$) |
| $A_0$ (fixed) | 15·5 | 9·2 | 4·1 | 1·7 | μb (GeV/$c$)$^{-2}$ |
| $A_1$ (fixed) | 3·95 | 2·34† | 1·28 | 3·4† | μb (GeV/$c$)$^{-2}$ |
| $B_0$ | 0·73 | 0·93 | 1·45 | 4·0 | (GeV/$c$)$^{-2}$ |
| $B_1$ | 0·73 | 0·9 | 1·45 | 3·2 | (GeV/$c$)$^{-2}$ |
| $R$ | 5·0 | 5·2‡ | 5·2 | 4·8‡ | (GeV/$c$)$^{-1}$ |

†(‡) Free (fixed) for ($\sigma_N$, $D$, $\sigma_-$, $P_-$) simultaneous fits at 6 GeV/$c$.

## 3. Conclusion

We have shown that the dual absorptive model for backward scattering can consistently explain the observed features of the πN differential cross section, polarization and the interference term. The model works very well for $N$ amplitudes for all $u > -1{\cdot}0$, while it fails to give correct Re $\Delta_1$; possibly Re $\Delta_0$ as well, for $u < -0{\cdot}6$.

The peripheral $N$ and $\Delta$ amplitudes have an interaction radius of $R \sim 1$ fm. $R_N$ varies very little with $s$ while $B_N$ increases with approximately ln $s$, which is compatible with standard Regge behaviour. Also the magnitude of $B_N$ is consistent with the corresponding forward value.

However, the slope parameter $B_\Delta$ is rather large and the resultant impact parameter distributions of Im $\Delta$ amplitudes are much less peripheral. This inconsistency is interpreted as due to the incorrect form of Re $\Delta_1$, as given by the model. In order to obtain a consistent value for the parameter $B_\Delta$, Re $\Delta_1$ must be very small or zero near $u \simeq -0{\cdot}5$. This can be achieved in a certain class of absorptive model where the imaginary parts are peripheral and the parameters $A$, $B$ and $R$ have particular non-Regge-type $s$ dependence. We can also achieve this kind of Re $\Delta$ in strong absorptive models.

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